Suppose you own an office complex with 100 offices available to rent. You can rent all of them if you set the rent at $800. You know that for each $10 increase in rent, one tenant will leave. What should the rent be in order to maximize your revenue? How many offices would be rented?

Revenue = (\# of offices \times \text{rental price})

\[ R(x) = (100 - x) \times (800 + 10x) \]

\[ R(x) = 80000 + 1000x - 800x - 10x^2 \]

\[ R(x) = 80000 + 200x - 10x^2 \quad \text{where} \quad 0 \leq x < 100 \]

\[ R'(x) = 200 - 20x \quad \text{(marginal revenue)} \]

\[ R'(x) = 0 \quad \text{OR} \quad R''(x) = -20 \]

\[ 200 - 20x = 0 \]
\[ -20x = -200 \]
\[ x = 10 \]

\[ \frac{-1}{10} \quad \frac{-1}{100} \]

\[ \text{max} \quad \text{at} \quad x = 10 \]

In order to maximize your revenue, 90 offices should be rented at a rental price of $900.
(27) \[ \text{profit} = \text{rev} - \text{cost} \quad x = \# \text{ of people on bus} \]

\[ \text{rev} = \left[ 200 - 2(x-50) \right]x = \left[ 200 - 2x + 100 \right]x = \left[ 300 - 2x \right]x = 300x - 2x^2 \]

\[ \text{cost} = 6000 + 32x \]

\[ P = 300x - 2x^2 - (6000 + 32x) \]

\[ P = -2x^2 + 268x - 6000 \]

\[ p'(x) = -4x + 268 \]

\[ p'(x) = 0 \]

\[-4x + 268 = 0 \]

\[-4x = -268 \]

\[ x = 67 \]

50 ≤ x ≤ 80

\[ \text{max when 67 people \at \text{profit}} \]

\[ p''(x) = -4 \]

\[ p''(67) = -4, \text{ cc } \downarrow, \text{ max} \]

50 + \frac{c}{2}

(50)

\[ n = \frac{a}{x-c} + b(100-x) \]

\[ \text{profit} = n \cdot (x-c) \]

\[ = \left[ \frac{a}{x-c} + b(100-x) \right](x-c) \]

\[ = a + b(100-x)(x-c) \]

\[ = a + b(100x - 100cx - x^2 + cx) \]

\[ P = a + 100bx - 100bcx - bx^2 + bcx \]

\[ p'(x) = 100b - 2bx + bc = 0 \]

\[-2bx = -100b - bc \]

\[ x = \frac{-100b - bc}{-2b} = 50 + \frac{c}{2} \]

\[ p''(x) = -2b \]

\[ p''(50+\frac{c}{2}) = \text{neg, cc } \downarrow \text{ max} \]

max profit at 50 + \frac{c}{2}